CALCULATION OF A TWO-PHASE FLOW IN AN AXISYMMETRIC NOZZLE

I. M. Kapura, V. I. Serebrennikov, and D. G. Chernikov

Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 5, pp. 631–638, 1967

UDC 532.529.5:533.601.15

A system of differential equations is presented to describe a two-phase flow through an axisymmetric supersonic nozzle. The results for the solution of this system on a "Minsk-14" computer are given. The theoretical results are compared qualitatively with the experimental data.

Papers devoted to the experimental and theoretical investigation of the process of gas discharge from a supersonic nozzle with condensed-phase inclusions are appearing with increasing frequency in the domestic and foreign literature [2, 3, 4]. It has been demonstrated in these references that the critical cross section for such a flow shifts to the diverging part of the nozzle, the condensed phase is left behind, and the velocity of the gas phase is reduced at the nozzle outlet in comparison with the flow of a pure gas. It had been ascertained experimentally that for the complete expansion of a gas with condensed particle inclusions a greater degree of nozzle expansion was needed than in the case of a pure gas.

Here we will calculate the process of a two-phase flow in an axisymmetric supersonic nozzle and examine certain of the relationships derived as a result of the calculation.

In deriving the equations describing the two-phase flow through an axisymmetric nozzle, we made the following assumptions:

1) the flow is steady and one-dimensional;

2) the heat capacity of the gas is independent of temperature;

3) the gravimetric composition of the two-phase flow does not change during the course of its motion;

4) the particles are spherical in shape and of identical diameter;

5) the specific volume of the particles is negligibly small in comparison with the specific volume of the gas phase;

6) the wave losses which become possible when the phase-velocity difference $w_g - w_s$ reaches the speed of sound are not taken into consideration;

7) the motion of the flow proceeds without exchange of heat with the ambient medium; losses due to the friction of the flow against the walls are not considered;

8) there is no exchange of heat between the solid particles and the gas.

With the assumptions made here, the process of a two-phase flow through an axisymmetric supersonic nozzle is described by the following equations:

the equation of state for the gas phase

$$\frac{1}{p}\frac{dp}{dx} - \frac{1}{\gamma_g}\frac{d\gamma_g}{dx} - \frac{1}{T}\frac{dT}{dx} = 0,$$

the continuity equation

$$\frac{1}{F}\frac{dF}{dx} + \frac{1}{w_g}\frac{dw_g}{dx} + \frac{1}{\gamma_g}\frac{d\gamma_g}{dx} = 0,$$

the equation of energy for the two-phase flow

$$w_g g_g \frac{dw_g}{dx} + w_g g_g \frac{dw_s}{dx} + c_p g_g \frac{dT}{dx} = 0, \qquad (1)$$

the equation of momentum

$$-\frac{g_g}{\gamma_g}\frac{dp}{dx} = g_g \frac{w_g}{g}\frac{dw_g}{dx} + g_s \frac{w_g}{g}\frac{dw_s}{dx},$$

the equation of motion for the solid particle

$$\frac{dw_g}{dx} = \frac{3}{4}\frac{c_x}{d_g}\frac{\gamma_g}{\gamma_s}\frac{(w_g - w_s)^2}{w_s}.$$

To integrate the system of differential equations (1) they must be brought to the form

$$\frac{dy_i}{dx} = f(x, y_1, y_2, \ldots, y_n) \quad i = 1, 2, \ldots, n. \qquad (2)$$

Having introduced the speed of sound, and having brought these equations to the form of (2), we obtain

$$\frac{dw_s}{dx} = \frac{3}{4}\frac{c_x}{d_s}\frac{\gamma_g}{\gamma_s}\frac{(w_g - w_s)^2}{w_s},$$

$$\frac{dw_g}{dx} = \frac{1}{M^2 - 1}\left(\frac{w_g}{F}\frac{dF}{dx} - \frac{g_s}{g_g} M^2 \frac{dw_s}{dx}\right),$$

$$\frac{dp}{dx} = -\frac{\gamma_g w_g}{g}\left(\frac{dw_g}{dx} + \frac{g_s}{g_g}\frac{dw_s}{dx}\right),$$

$$\frac{dT}{dx} = -\frac{w_g}{c_p}\left(\frac{dw_g}{dx} + \frac{w_s}{w_g}\frac{g_s}{g_g}\frac{dw_s}{dx}\right),$$

$$\frac{d\gamma_g}{dx} = \frac{\gamma_g}{p}\frac{dp}{dx} - \frac{\gamma_g}{T}\frac{dT}{dx}. \qquad (3)$$

The second equation in system (3) represents the so-called condition of inverse action. Vulis [1] established a relation similar to this for a pure gas.

As follows from the second equation of system (3), two actions are imposed simultaneously on a gas: the geometric $\frac{w_g}{F}\frac{dF}{dx}$ and the mechanical (the friction of the gas against the particles and the expenditure of kinetic energy to accelerate the particles) $\frac{g_s}{g_g} M^2 \frac{dw_s}{dx}$.

It is this circumstance that serves as the basis for all phenomena occurring in a supersonic nozzle through which a two-phase flow is proceeding. The sign of the quantity $\frac{1}{M^2 - 1}$ varies on transition of the velocity through the critical value and the nature of the influence exerted by the individual physical actions on the gas flow is therefore different (opposite) for subsonic and supersonic flows. The critical velocity can be attained only in the expanding part of the nozzle, since

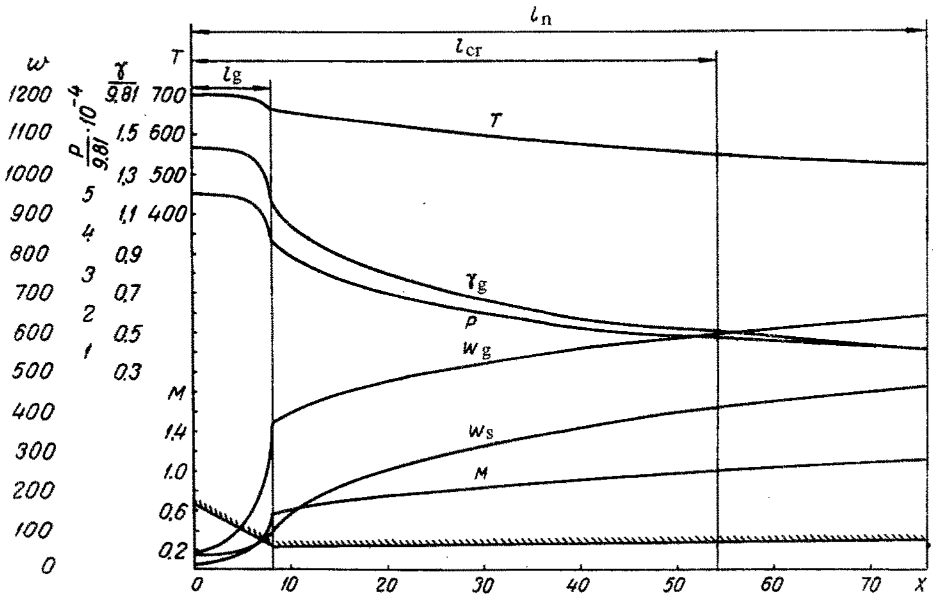

Fig. 1. Change in two-phase flow parameters T(°K), γ_g (N/m^3), p (N/m^2), w_g (m/sec), w_s (m/sec), M with respect to the nozzle length x (mm) for weight composition g_g = = 0.4 and particle size $d_s = 10^{-5}$ m.

$M=1$ for $\frac{w_g}{F}\frac{dF}{dx} = \frac{g_s}{g_g} M^2 \frac{dw_s}{dx}$. In other words, transition through the critical velocity is possible only at the point at which $\frac{w_g}{F}\frac{dF}{dx} - \frac{g_s}{g_g} M^2 \frac{dw_s}{dx} = 0$, while the velocity is subsonic in the narrow section of the nozzle in this case. On the basis of the form of the expression in the parentheses we can state that if the particles are small in diameter and given that they are present in great number, the downstream shift in the critical cross section should be greater than in the case in which the particles are large and few in number.

To solve the system of differential equations (3) we must assume the boundary conditions for five variables (p_0, T_0, γ_{g0}, w_{g0}, w_{s0}). If the nozzle were converging (a subcritical pressure difference), all five parameters could be specified at the nozzle inlet and the problem would be solved. However, if the nozzle profile is supercritical (a de Laval-type nozzle), additional considerations are needed for the solution of the problem. Since the critical parameters in a supersonic nozzle may be obtained only where $\frac{w_g}{F}\frac{dF}{dx} - \frac{g_s}{g_g} M^2 \frac{dw_s}{dx} = 0$, the boundary condition for the gas velocity may be specified only in the form

$$w_g = a \quad \text{for} \quad \frac{w_g}{F}\frac{dF}{dx} = \frac{g_s}{g_g} M^2 \frac{dw_s}{dx}.$$

It is precisely here that the critical cross section of the nozzle will be found.

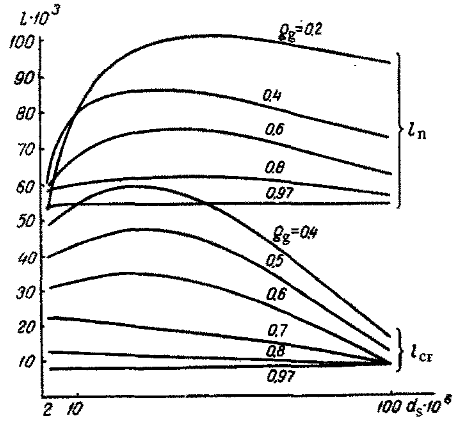

Fig. 2. Distance to critical section l_{cr} (m) and nozzle length l_n (m) as functions of particle size d_s (m) and weight composition g_g.

The system of ordinary differential equations (3) was solved by the Runge-Kutta method on a "Minsk-14" computer. The nozzle profile was given before the integration was started. For convenience in comparing the results of the various calculations, all of the nozzles were of identical throat area and the expanding sections of the nozzle all had an identical divergence angle. The area of the nozzle outlet was determined by integration to complete expansion of the gas (up to $p = 0.981 \cdot 10^5$ N/m²). The nozzle profile was assumed to be rectilinear (dr/dx = const). The velocity of the solid particles at the inlet to the nozzle was $w_{g0} = 0.9 w_{g0}$. The main calculation was carried out for the following initial data: $p_0 = 4.91 \cdot 10^5$ N/m², $T_0 = 700°$ K; $g_g = 0.97$; 0.8; 0.6; 0.4; 0.2; $d_s = 10 \cdot 10^{-6}$m; 10^{-6}m; $0.2 \cdot 10^{-6}$m; $\gamma_s = 2.94 \cdot 10^4$N/m³; k = 1.3; C_p = 2100 J/kg · deg; R = 491 J/kg · deg; $\mu_g = 3.92 \cdot 10^{-6}$ N · sec/m²; g = 9.81 m/sec².

Figure 1 shows the change in the parameters p, T, γ_g, w_g, w_s, and M along the nozzle length for $g_g = 0.4$; $d_g = 10 \cdot 10^{-6}$ m. As we can see from the figure, in the narrow nozzle section (x = 8 mm) a gas velocity equal to the speed of sound is not attained and the critical cross section shifts to the expanding part (x = 54 mm).

Figure 2 shows how the critical cross section of the nozzle shifts for various g_g and d_s. With a reduction in g_g the critical cross section shifts all the more markedly to the expanding part. For $g_g = 0.2$, there is no function $l_{cr} = f(g_g, d_s)$, since the pressure difference $p_0/p_1 = 5$ was inadequate to accelerate the gas to M = 1. When $g_g = 0.97$ the critical cross section is found virtually at the narrow cross section of the nozzle.

While the weight composition affects the magnitude of the shift in the critical cross section in a completely defined manner, the same cannot be said of the particle dimensions. For each g_g there is a completely defined particle dimension for which the magnitude of the shift attains a maximum value. A shift to the left and to the right of the maximum causes the two-phase flow to approach a pure gas. A shift to the left results from a reduction in the velocity difference between the gaseous and solid phases and approach to total kinematic equilibrium (here there is a reduction in the mechanical action as a result of a reduction in the irreversible losses due to friction), while a shift to the right results from a reduction in the flow of energy to accelerate the particles because of a pronounced drop in the velocity of the particles as their diameters are increased.

The length of the nozzle is affected analogously by g_g and d_s. With an increase in the total mechanical action (the mechanical losses plus the energy to accelerate the particles), the nozzle length increases, reaching a maximum value for a certain d_s. The reduction in g_g at all particle diameters leads to an increase in nozzle length, since with a reduction in g_g the mechanical action on the gas is increased. The curve for $g_g = 0.2$ is somewhat different in nature. This is explained by the fact that for $d_g = 0.2 \times 10^{-6}$ m and 10^{-6} m the pressure difference was inadequate to accelerate the gas to the speed of sound, since a great portion of the energy flux was expended on particle acceleration. It should also be noted that if we take a nozzle whose length is equal to that of a nozzle calculated for a pure gas, and if a two-phase flow were passed through such a nozzle with the same pressure difference, the nozzle would function with underexpansion. Thus, for $g_g = 0.4$ and $d_s = 10^{-6}$m (see Fig. 1) the pressure at the point corresponding to the length of such a nozzle (x = 48.6 mm) amounts to $p = 1.42 \cdot 10^5$ N/m². The same result was obtained by Komov by direct measurement of the pressure at the nozzle outlet in the flow of a pure gas through that nozzle and for the flow of a gas with condensed particles [2]. The same reference presents photographs of the compression shocks behind the nozzle outlet. The compression shock in a two-phase flow is situated farther upstream than in the case of the discharge of pure air. Since the critical cross section of a two-phase flow shifts to the expanding part of the nozzle, the velocities of the gaseous phase of a two-phase flow and of a pure gas at the nozzle outlet will be different, with the pure gas exhibiting a higher velocity. This obviously results in a situation in which the compression shock in the two-phase flow will shift farther upstream than the compression shock in a pure gas.

The analysis of the changes in the velocities of the gas and solid particles as a function of the change in the weight composition and particle dimensions (Fig. 3a) shows that the largest particles exhibit the greatest lag, while the least lag is exhibited by the fine particles. It is interesting to note that large particles over the entire range of variation in g_g change their velocity only slightly. The velocity of the gas and of the particles increases with increasing g_g, while the gas velocity for g_g close to 1 tends to the velocity of discharge for a pure gas.

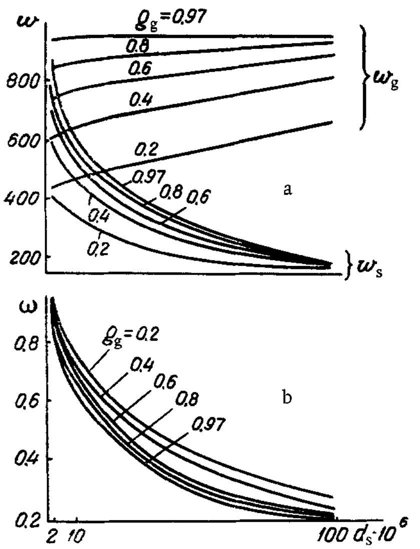

Fig. 3. Change in gas velocity w_g (m/sec), solid particles w_s (m/sec), and slip coefficient ω as functions of a change in particle size d_s (m) and weight composition g_g.

To analyze the irreversible losses arising as a result of particle friction against the gas, it is of interest to trace the change in the adiabatic efficiency η of the discharge process for a two-phase flow. This can be expressed as

$$\eta = \frac{T_0 - T_2}{T_0 - T_1}. \qquad (4)$$

However, the same efficiency can be expressed in terms of the phase velocities, using the equation for the conservation of energy

$$\eta = \frac{w_g^2 + \frac{g_s}{g_g} w_s^2}{w_{ag}^2}.$$

Since $\frac{T_1}{T_0} = \left(\frac{p_1}{p_0}\right)^{\frac{k-1}{k}}$, with consideration of (4)

$$\frac{T_2}{T_0} = \eta \left(\frac{p_1}{p_0}\right)^{\frac{k-1}{k}} + (1 - \eta).$$

From the function $\eta = f(g_g, d_s)$, shown in Fig. 4a, we see that with a reduction in g_g the irreversible los-

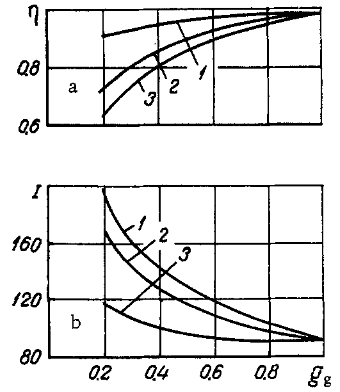

Fig. 4. Dependence of efficiency η of outflow and specific impulse I (sec) of two-phase flow related to a gas phase on weight composition g_g and particle size d_s (m). 1) $d_s = 0.2 \cdot 10^{-5}$; 2) 10^{-5}; 3) $10 \cdot 10^{-5}$.

ses due to friction increase and the adiabatic efficiency of the discharge process for the two-phase flow diminishes. With a reduction in particle dimensions these lag decreasingly behind the gas and the losses due to friction diminish. The losses are reduced particularly sharply when the particle dimensions become smaller than $(1-2) \times 10^{-5}$ m.

It has been established that with increasing particle dimension the slip factor $\omega = w_s/w_g$ (Fig. 3b) diminishes. The dependence of the slip factor on the weight composition is rather weak in comparison with its dependence on the particle dimensions. This is particularly valid for small particles. Thus, for particles with $d_g = 0.2 \cdot 10^{-5}$ m with a change in g_g from 0.97 to 0.2 the slip factor changes from 0.93 to 0.95. The increase in the slip factor with an increase in the weight composition of the solid phase can be explained by the increase in nozzle length in this case (see Fig. 2). With a greater nozzle length the particles can achieve greater acceleration and the slip factor increases. An increase in particle lag leads to a pronounced increase in the irreversible friction losses. It is precisely for this reason that the greatest losses occur in the motion of a two-phase flow with large particles (Fig. 4a).

The specific impulse of a two-phase flow (Fig. 4b), referred to the gas phase,

$$I = \frac{w_g}{g} + \frac{g_s}{g_g} \frac{w_s}{g}.$$

As we can see from the figure, with an increase in the weight composition of the solid phase the specific impulse increases. A reduction in particle dimensions leads to an even greater increase in the specific impulse. The specific impulse of a two-phase mixture for $g_g = 0.2$ and $d_s = 0.2 \cdot 10^{-5}$ m is greater than the specific impulse for a pure gas by a factor of approximately two. For large particles $d_s = 10 \cdot 10^{-5}$ m in the range $g_g = 0.5–1$ the specific impulse remains virtually unchanged and is approximately equal to the specific impulse of a pure gas, which can be explained by the virtually constant and extremely low velocity of the solid phase.

It is obvious that the results of the calculations are limited by the framework of the adopted assumptions. In conclusion, we should dwell on one of these, i. e., on the assumption that there is no exchange of heat between the phases. To evaluate the effect of this assumption on the flow parameters at the nozzle outlet, we carried out a calculation with consideration of the transfer of heat between the phases for $g_g = 0.4$ and $d_s = 10^{-5}$ m. After the calculation was carried out the heat-transfer equation

$$\frac{dT_s}{dx} = -\frac{6g\alpha}{c_s d_s \gamma_s w_s}(T_s - T_g)\frac{1}{3600},$$

was introduced into the system of equations (3) and the equation for the conservation of energy will have the form

$$\frac{dT_g}{dx} = -\frac{w_g}{c_p}\left(\frac{dw_g}{dx} + \right.$$

$$\left. + \frac{g_s}{g_g}\frac{w_s}{w_g}\frac{dw_s}{dx}\right) - \frac{g_s c_s}{g_g c_p}\frac{dT_s}{dx}.$$

In the determination of the heat-transfer coefficient α it was assumed that the spherical particles are subject to steady streamlining. This same assumption served to evaluate the maximum effect which might be exerted, by the exchange of heat between the phases, on the process for the discharge of two-phase flow. As a matter of actual fact, the flow past the spheres is nonsteady and the exchange of heat between the phases is smaller.

The calculation showed that consideration of the exchange of heat (for $g_g = 0.4$ and $d_s = 10^{-5}$ m) leads to an insignificant change in the efficiency of the discharge process (by 0.3%) and in the specific impulse (by 0.15%).

This slight effect on the part of heat transfer may be explained by the fact that the heating of the gas due to the transfer of heat along the nozzle occurs most intensively at the end of the expansion process.

NOTATION

d_s is the solid-particle diameter;

g_g and g_s are the mass fractions of the gas and solid phases in a flow;

w_g and w_s are the velocities of the gas and solid phases in a flow;

ω is the slip coefficient;

a is the speed of sound of the gas phase;

p is the pressure of the gas phase in flow;

γ_g and γ_s are the specific weight of the gas and solid phases in the flow;

r and F are the radius and area of a nozzle cross-section;

x is the coordinate along the nozzle axis;

y is the normal to it;

l_{th}, l_{cr}, and l_n are the distances from the exit to the throat, to the critical cross-section, and to the nozzle exit;

c_x is the resistance coefficient of a sphere placed into a gas flow;

M is the relation of the velocity of a gas phase to the speed of sound;

k, c_p, R are the adiabatic exponent, the specific heat capacity at constant pressure, and the gas constant of the gas phase;

μ_g is the dynamic viscosity coefficient of the gas phase;

T_0 is the gas temperature at the nozzle exit;

T_1 is the gas temperature at the nozzle outlet when pure gas flows over it under adiabatic conditions;

T_2 is the gas temperature at the nozzle outlet when two-phase flow moves over it (for an identical pressure drop);

η is the adiabatic efficiency of two-phase discharge;

I is the specific impulse related to the gas phase;

α is the heat transfer coefficient;

Subscripts: 0 is the parameter at the nozzle exit; 1 is the parameter at the nozzle outlet; g is the gas phase; s is the solid phase; th is the throat; cr is the critical section; n is the nozzle.

REFERENCES

1. L. A. Vulis, DAN SSSR, **54**, no. 8, 1946.

2. G. A. Komov, IFZh [Journal of Engineering Physics], **10**, no. 3, 1966.

3. Detonation and Two-Phase Flow [Russian translation], ed. S. S. Penner and F. A. Williams, Academic Press, 1966.

4. Kligel, Voprosy raketnoi tekhniki, no. 10, 1965.

4 April 1967

All-Union Correspondence
Polytechnic Institute, Moscow